Inclusive mathematics teaching: supporting those that struggle

The Trouble with Maths!

Judy Sayers PhD, FHEA
Stockholm University

JUDY SAYERS

- Teacher
- Special needs teacher
- Gifted and talented mathematics teacher
- Teacher trainer in special educational needs

- Special educational needs teacher in school
- Interest in psychology and difficulties children have in learning mathematics
- Why I became a researcher

TEACHING MATHEMATICS

The ability of students to learn mathematics with understanding relies on a classroom community in which all students are expected to learn through active participation, and teachers provide support to engage all students in mathematical tasks.

Bowe, F. (2005). *Making inclusion work*. Prentice Hall.

Stockholms universitet

HOW DO **WE** ATTEMPT TO MEET THE AIMS OF THE CURRICULUM?

- Professional development to increase regular classroom teachers' knowledge of educating exceptional students
- Careful planning and instructional design
- Use technology to allow subject matter to be accessible in a variety of formats

- OTHER Challenges?

Stockholms universitet

HOWEVER…
SOME CHALLENGES CAN BE STUDENTS WHO EXHIBIT BARRIERS IN THEIR LEARNING – WHO ARE THEY?

- Students for which Swedish is a second language
- Students with physical disabilities
- Students with learning disabilities
- Gifted students
- Students with non-traditional learning approaches
- All students!

Stockholms universitet

IDENTIFYING AREAS OF INDIVIDUAL NEED

Cognition and learning

- Specific learning difficulties (SpId) (specifika inlärningssvårigheter)
- Moderate learning difficulties (MLD)
- Severe learning difficulties (SLD)
- Profound multiple learning difficulties (PMLD)

Behaviour, Emotional and Social Development

- Behaviour, Emotional and Social difficulty (BESD) (Ledam)

Communication and Interaction Needs

- Speech, language and communication needs (SLCN)
- Autistic Spectrum Disorder (ASD) (Autistisk)

Sensory and/or Physical needs

- Hearing or visual impairment (HI or VI)
- Physical disability (PD)
- Muliti-sensory Impairment (MSI)

KEY FOCUS OF THIS SESSION:

Students who exhibit difficulties in:

- Memory and general strategy use
- Specific processes and strategies associated with mathematics problems
- Language and Communication
- Low motivation and affect

- These students also need to be included in the lesson.

Chinn (2010)

Stockholms universitet

INCLUSION
SOME POINTS FOR DISCUSSION

1. Planning for inclusion should be an **integral** part of a **wider, coherent approach** to effective mathematics planning.
2. Making mathematics more accessible for some pupils can have a **positive benefit for all** pupils.
3. Where pupils are not given the opportunity to understand mathematics and develop confidence then mathematics teaching is not inclusive and pupils are denied full access to the mathematics curriculum.
4. **Extracting pupils** from their *mainstream* (class) environment in order to help them 'catch up' is **not a good model** of inclusion and can do harm.

Chinn (2010)

Stockholms universitet

1. COHERENT APPROACH

- *Planning for inclusion should be an **integral** part of a **wider, coherent approach** to effective mathematics planning.*

1. Whole school approaches are most effective…
   - All teachers use same approach to support
   - *alla lärare använder samma strategi för att stödja lärande*

   - Dimmock, C. (2013). *School-based management and school effectiveness*. Routledge.

Chinn (2010)

Stockholms universitet

2. APPROACH: PHILOSOPHIES OF EDUCATION

- Existentialism
  - *Free will (fri vilja)* to develop as the student sees fit, self-responsibility, play is “good”: Sartre
- Behaviorism
  - Change the environment *(förändring miljö)*, change the student—teach scientifically: Pavlov, Skinner,
  - Modern perspective on IQ (entity or incremental)
    - Theory of motivation (Dweck, 1999)

Stockholms universitet

CONSEQUENCES OF DWECK'S THEORY OF MOTIVATION -FOR TEACHERS

Recommends teachers to…

Avoid giving ‘person-orientated praise':

- e.g. ‘I'm proud of you'; ‘you're good at this'. Because it:
- assumes that success is due to personal attributes.
- teaches students to interpret difficulties in terms of their personal weaknesses.

Dweck, C. S., Mangels, J. A., Good, C., Dai, D. Y., & Sternberg, R. J. (2004). Motivational effects on attention, cognition, and performance. *Motivation, emotion, and cognition: Integrative perspectives on intellectual functioning and development*, 2, 41-55.

Stockholms universitet

MOTIVATION CONT.

Instead, give ‘process orientated praise'. This is focussed on the process required for success. For example, praise the student's effort and strategy .

- E.g: ‘You really tried hard'; ‘That was a good way to do it'; Because it: sells the idea that esteem comes from striving and from the use of effective strategies.
- teaches students to interpret setbacks in terms of lack of effort, or inappropriate strategies.
- allows every student to earn praise.

Use also ‘task orientated' praise .

For example:

- ‘All the labels are correct'
- ‘There are hardly any spelling mistakes this time.'

Chinn (2010)

Stockholms universitet

2. ACCESSIBILITY

- *Making mathematics more accessible for some pupils can have a **positive benefit for all** pupils.*

Chinn (2010)

Stockholms universitet

COGNITIVISM: LEARNING THEORY

- Atkinson-Shriffin and Ausubel
  - Schema, Scaffolding, Chunking, Advanced Organiser
  - Three-Stage Information Processing
    - Sensory Registry (1/4 second)
    - Short-term Memory (18 seconds)
    - Long-term Memory (unlimited)

Stockholms universitet

CONSTRUCTIVIST THEORIES...

Constructivist and social constructivist Theory

- Build on Prior Knowledge
- Construct New Knowledge
- Understand through Authentic Experiences
  - Active, hands-on
- Problem Solving, Exploration, Collaboration
- Revision and Reflection

Jerome Bruner

- Learning is an active process
- Students build new ideas based on their existing knowledge
- Discovery Learning
- Spiral Curriculum

Stockholms universitet

3. OPPORTUNITIES TO BUILD CONFIDENCE

- *Where pupils are not given the opportunity to understand mathematics and **develop confidence** then mathematics teaching is not inclusive and pupils are denied full access to the mathematics curriculum.*

- BUT… will that be just the slower learners?

Chinn (2010)

Stockholms universitet

GIFTED STUDENTS IN MATHEMATICS

How Gifted Learners Differ from Classmates	Relationship to Mathematics Learning
1. Pace at which they learn	1. The sequential nature of math content makes pacing an issue.
2. Depth of their understanding	2. Deeper levels of understanding and abstraction are possible for most mathematical topics, so differentiation becomes important.
3. Interests that they hold (Maker, 1982)	3. If the interest is snuffed out early, the talent may not be developed.

- Inclusion vs Exclusion of gifted students from classrooms

- Eyre, D., & Lowe, H. (2013). *Curriculum provision for the gifted and talented in the secondary school*. Routledge.

Stockholms universitet

SPECIAL LEARNING IN MATHEMATICS

- This term covers a range of frequently co-occurring difficulties, more commonly:
- Dyslexia. *dyslexi*
  Dyspraxia/DCD.
  Klappar
- Dyscalculia.
  Dyskalkyli
- Attention Deficit Disorder (A.D.D)/Attention Deficit Hyperactivity Disorder (A.D.H.D.)
  Auditory Processing Disorder.

Stockholms universitet

4. EXTRACTING STUDENTS FROM THE CLASSROOM

- ***Extracting pupils*** *from their (mainstream/class)* ***environment*** *in order to help them 'catch up' is* ***not a good model*** *of inclusion and can do harm.*

- Bart: Let me get this straight: we're behind the rest of our class and we're going to catch up to them by going slower than they are? … *Cool*!

Stockholms universitet

WHAT IS IN FOCUS FOR A TEACHER TO ACCOMMODATE STUDENTS WITH 'SPECIAL NEEDS' IN MATHEMATICS?

- Specific subject matter?
- All the subject matter?
- The pace of the curriculum?
- The didactics?
- The teachers?
- The culture of mathematics and/or the classroom?
- The beliefs: explicit or ingrained?

- Is it the learner, or the way they are taught?

Stockholms universitet

Where does that leave us?

Let us see what some of the experts (or self professed experts) say, but we should be mindful of what we *just* accept

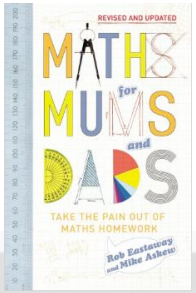
Chinn (2010)

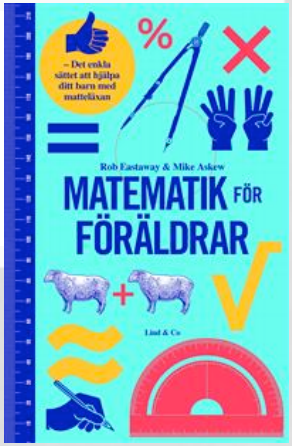
Stockholms universitet

MULTIPLE INTELLIGENCES AND SPECIFIC LEARNING DIFFICULTIES/ DIFFERENCES

- Gardner's theory of multiple intelligences suggests that there are ~~8~~ 9 intelligences (for example, linguistic, logical, musical).
- We do not necessarily have equal abilities in all these intelligences, If you are
  logico-mathematical in
  as having a SpLD. Not

SOME OTHER ISSUES





- Rob Eastaway (2010): 'Mismatch between methods that parents know and those taught in primary schools'.
- Jo Boaler: 'It's about having methods demonstrated and reproducing them – giving children a very narrow view of maths.
- The curriculum has a lot of methods and teachers respond with having to teach method after method.'

Boaler, J. (2009). *The Elephant in the Classroom. Helping Children Learn and Love Maths.* *London: Souvenir Press.*

'Is underachievement restricted to low achievers?'

Interventions should take into account that mathematical ability is not a single entity.' (2004)

Dowker, A. (2004) *What works for Children with Mathematical Difficulties?* DfES: Nottingham.

Stockholms universitet

ATTITUDE AND MOTIVATION

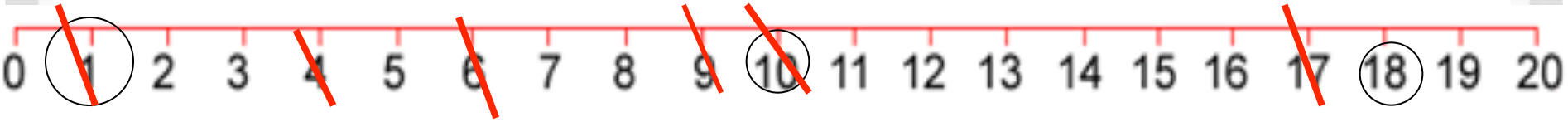
- Singapore mathematics reported to have 95% of all students achieve high level scores in their mathematics, the 5% '*do not want to learn*'

  Berinderjeet Kaur, (2015) Mathematics & Mathematics Education Academic Group, National Institute of Education, Singapore.

- How has this developed in society?

Chinn (2010)

Stockholms universitet



6 + 4 = 10

10 take away 9 makes 1

1 add 17 is 18

18……

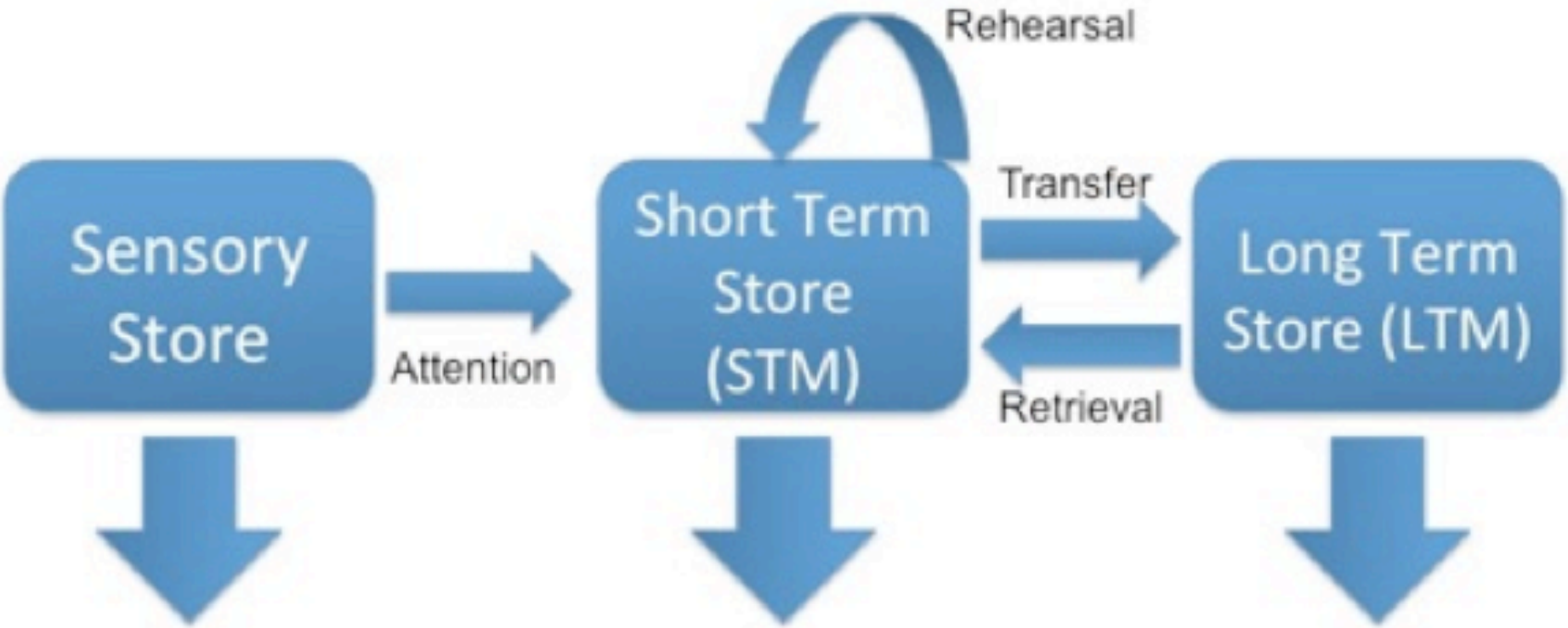
Competitive aim – stop your partner from going

Collaborative aim – cross off as many as possible

Stockholms universitet

SHORT TERM MEMORY AND WORKING MEMORY

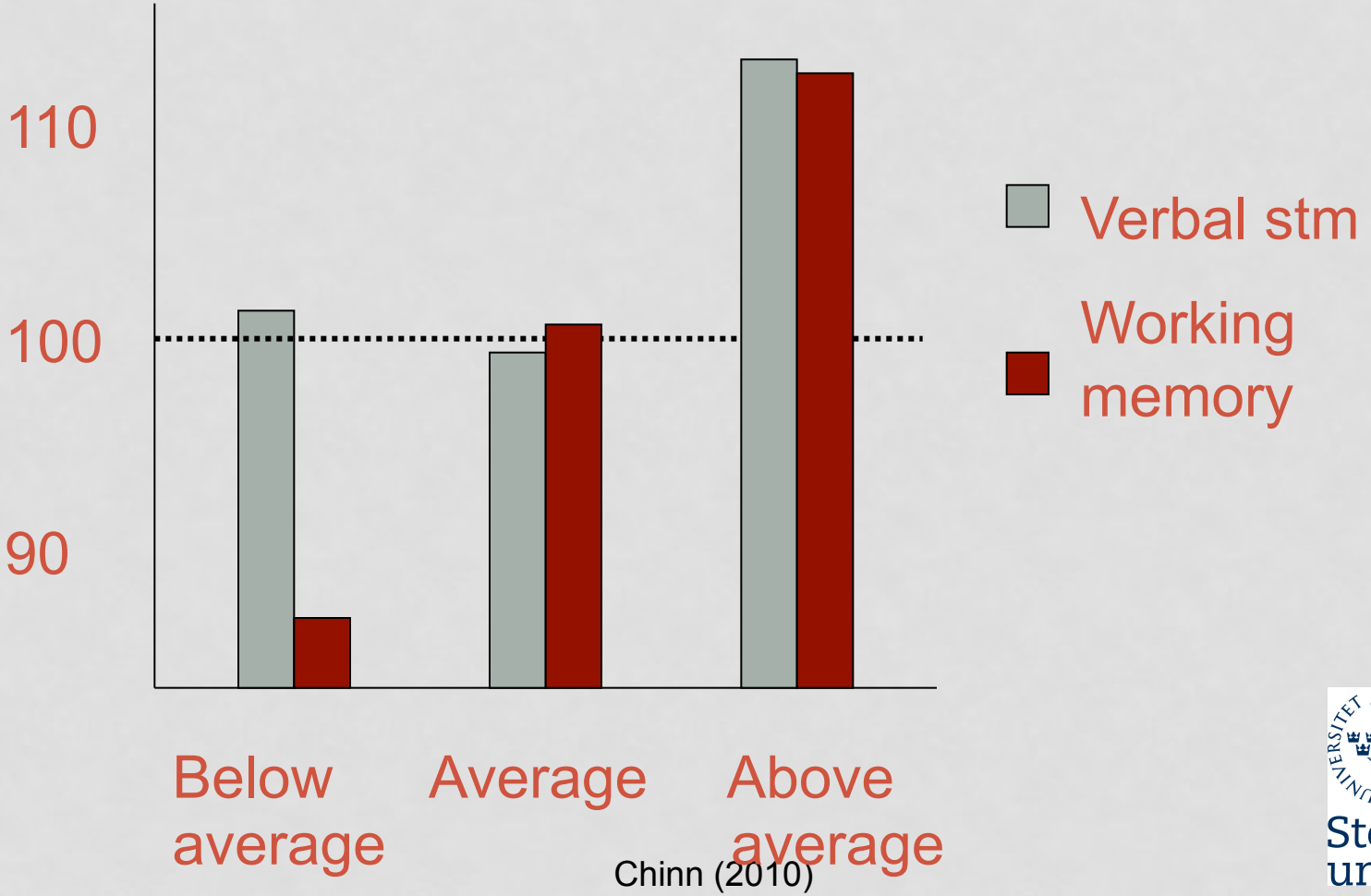
(Atkinson & Shiffrin, 1968)



Chinn (2010)

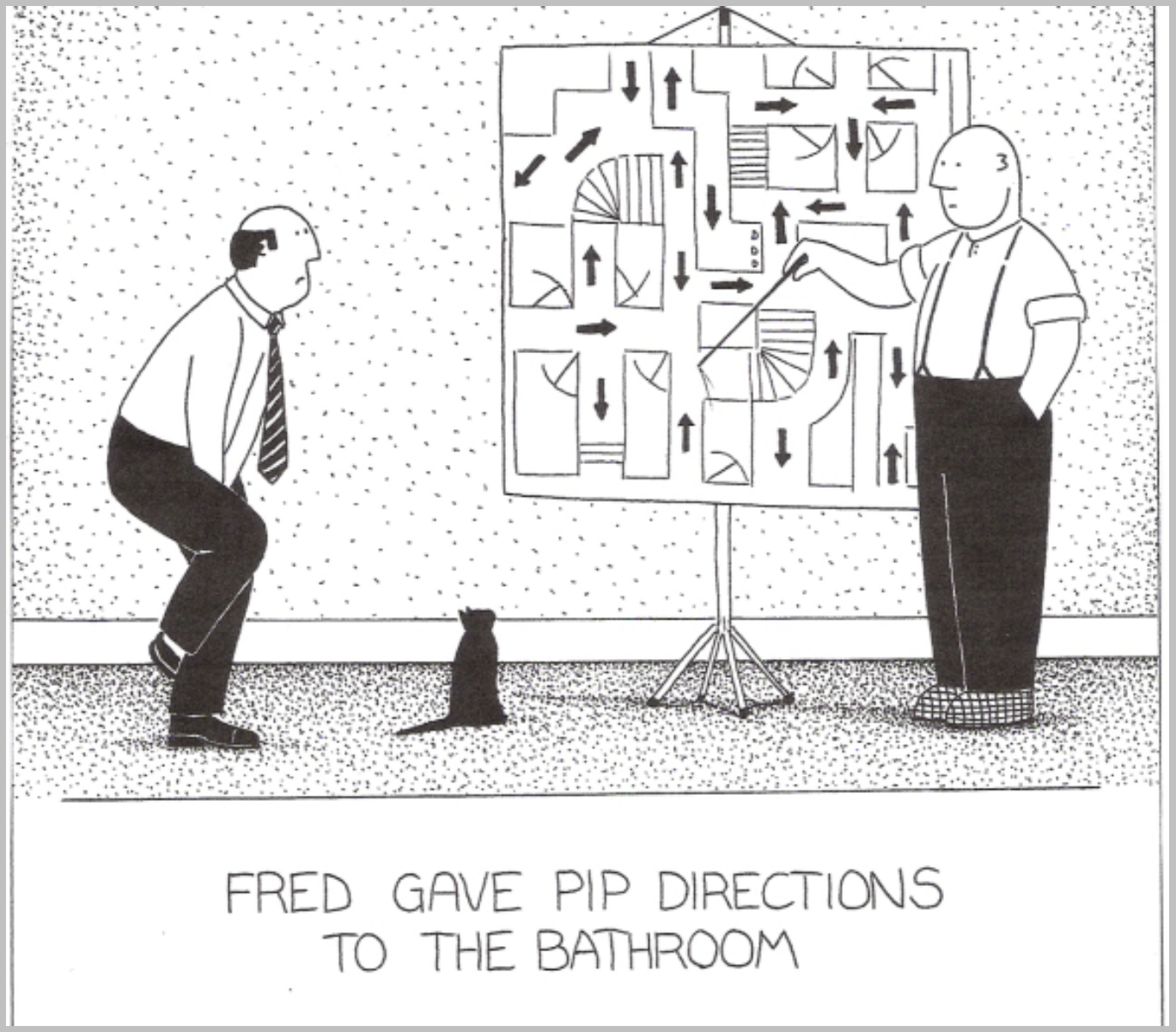
Stockholms universitet

STM, WORKING MEMORY AND MATHEMATICS IN 6 AND 7 YEAR OLDS



Chinn (2010)

Stockholms universitet



Chinn (2010)

LONG TERM MATHEMATICAL MEMORY RETRIEVAL OF BASIC FACTS

- children aged between 8.2 to 9.4 years, became faster and more accurate at solving math problems,
- relied more on retrieving facts from memory and less on counting.
- **As these shifts in strategy took place, the researchers saw several changes in the children's brains.**
- The hippocampus, a region with many roles in shaping new memories, was activated more in children's brains after one year. Regions involved in counting, including parts of the prefrontal and parietal cortex, were activated less.

Teachers still have a responsibility to support these types of changes taking place

Menon et al. 2014, Stamford psychiatry & Behavioural sciences

Stockholms universitet

Definition

Dyscalculia is:

…a condition which affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures.

Mahesh Sharma - Definition

Dyscalculia is:

A disorder in the ability to do or to learn mathematics ie. Difficulty in number conceptualisation, understanding number relationships, and difficulty in learning algorithms and applying them. (1990)

The dyscalculic individual may have sufficient intellectual ability and proper motivation, yet, the individual will show lower than average mathematical age in relation to normal mental age. (1990)

Dyscalculia

Incidence

About 7% of children have some form of learning disability in mathematics

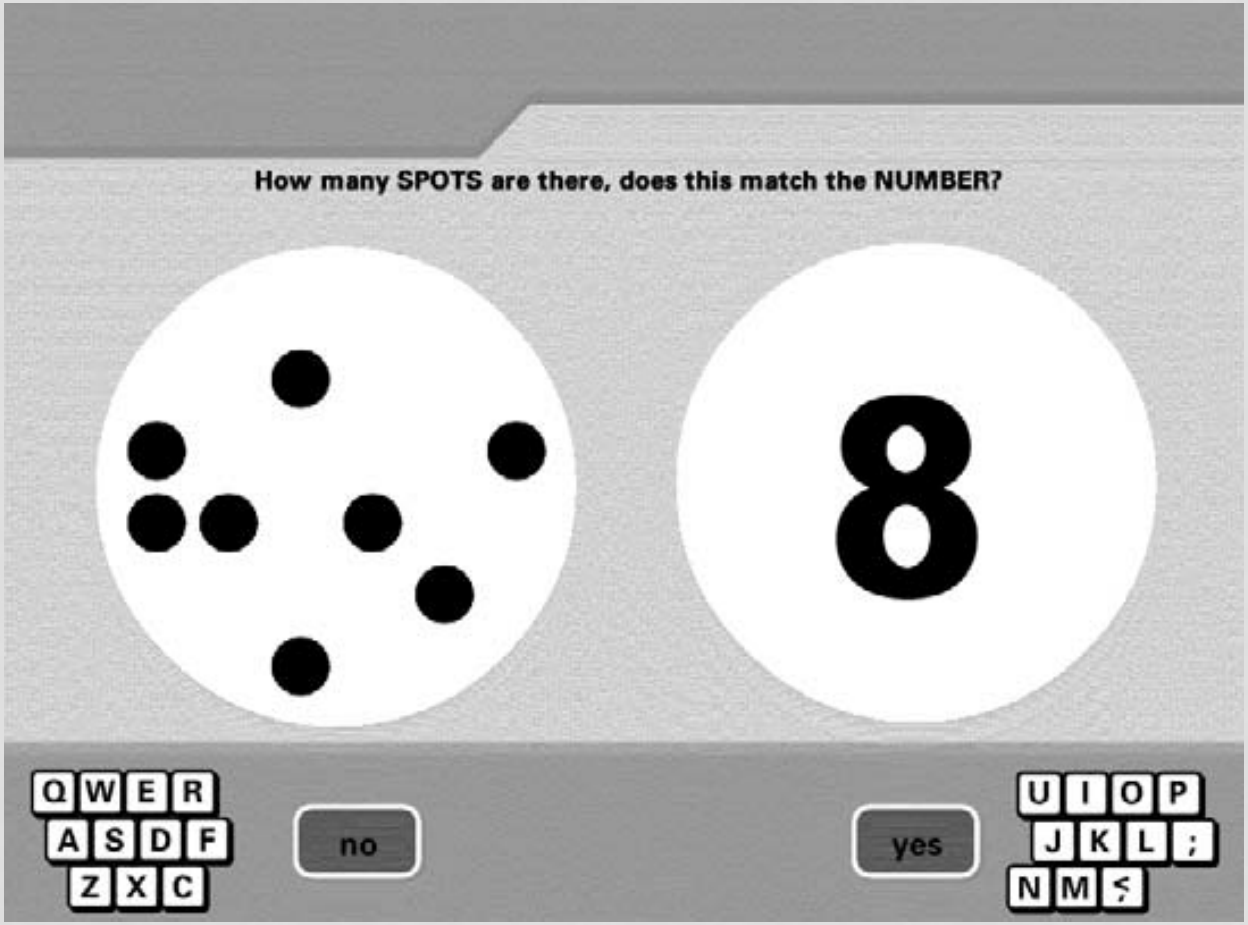
(Geary, 1996)

4-6% of the population have dyscalculia

(Butterworth, 1999)

Dyslexia		Dyscalculia
Genetic Brain-based • structure • cerebellum • cellular	Biological	Genetic Brain-based • number module
• phonological processing • working memory • visual processing	Cognitive	• numerosities • subitising • working memory for numerical information
Counting, arithmetic, remembering number facts and procedures, understanding concepts	Behavioural	Counting, arithmetic, remembering number facts and procedures, Understanding concepts

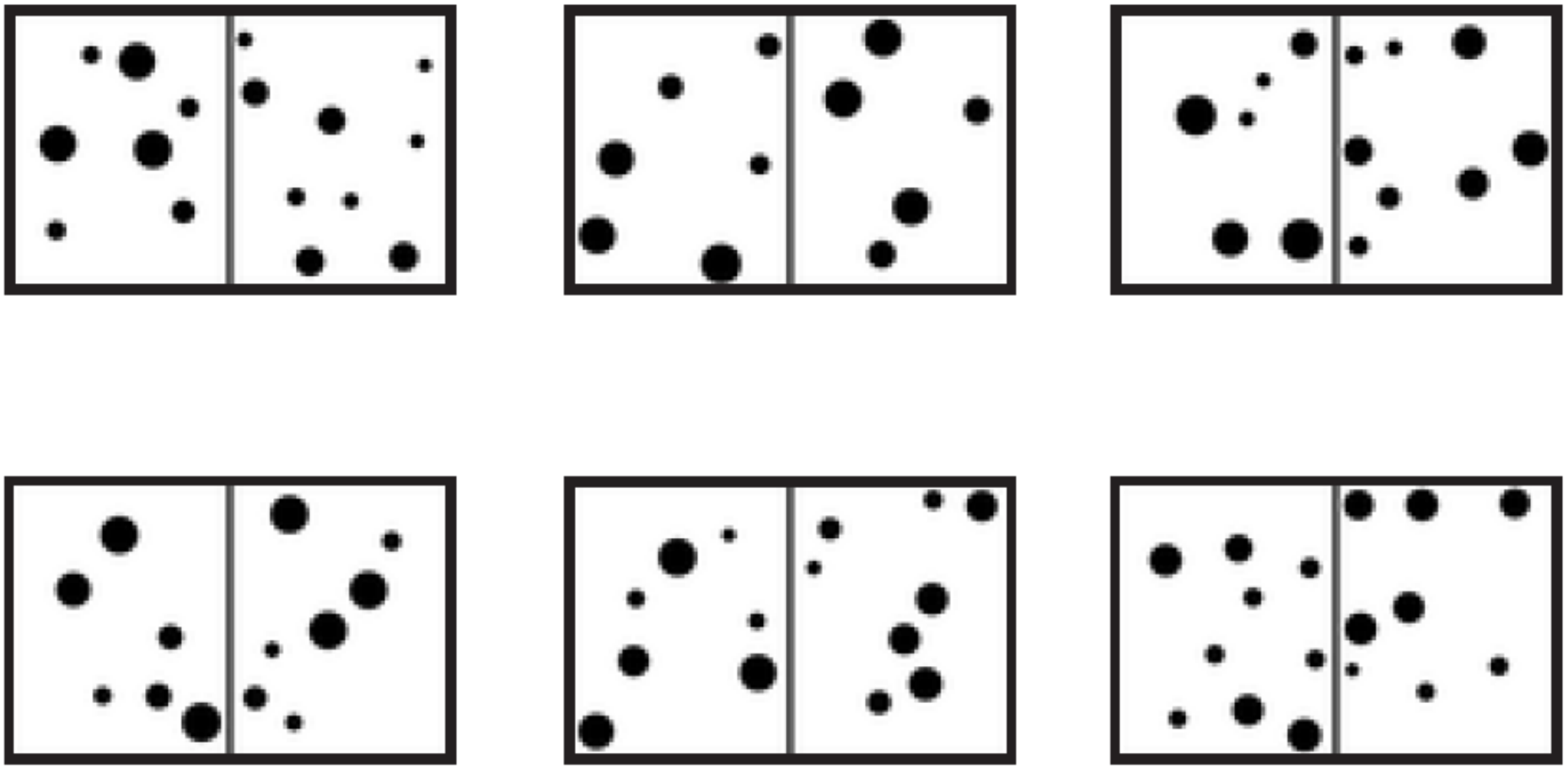
BUTTERWORTH'S DYSCALCULIA SCREENER



Chinn (2010)

Stockholms universitet

Numeracy screener (Ansari et al, Canada)



1	9

8	1

7	1

9	2

3	7

7	2

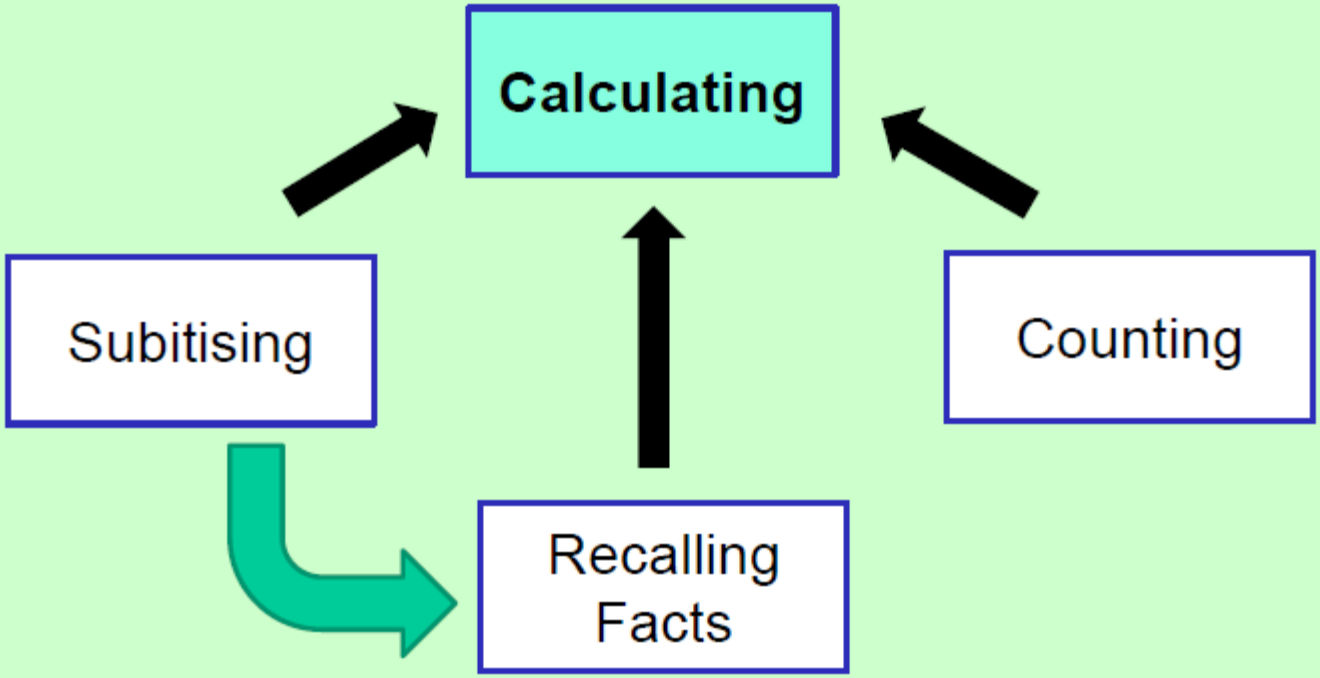
2	5

8	2

Ansari et al. Canada, Numeracy screener)

DOES A SCREENER HELP TEACHERS?

- *Kanske!*
- Dyscalculia might have been misdiagnosed by the Screener, as other studies have found. Messenger et al (2007) found that some high achievers were identified as dyscalculic, while others were not, whom teachers thought more likely. Some children originally identified as dyscalculic were later no longer so identified, without any intervention.
- Voutsina and Ismail (2007) reported that the Screener ignored children's understanding and individual strategies, so that children with less understanding scored better than those with good understanding whose responses were slow. The Dyscalculia Screener also identified children as dyscalculia who pressed the wrong keys but said the right answer.
- Dyscalculia is identified by percentile rather than characteristics of an impairment. Caution is needed in using.

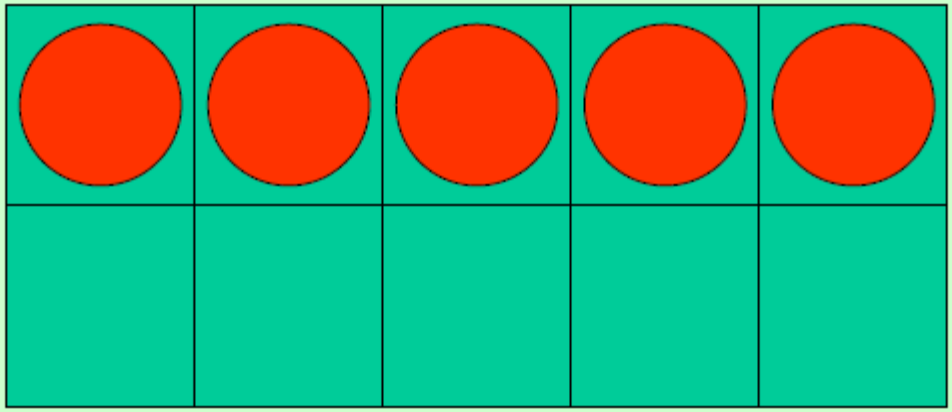

Calculating
Subitising
Counting
Recalling
Facts

LEARNING FROM OTHERS…

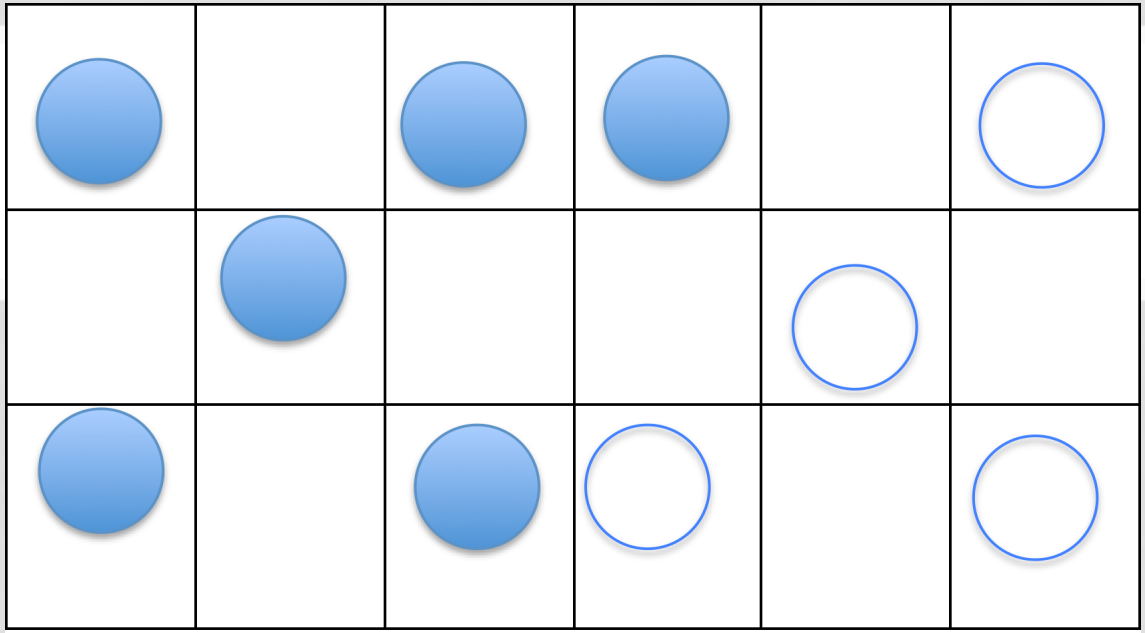
- Other countries approaches to the teaching of mathematics appear to have a more positive outcome for those students who struggle. Why?

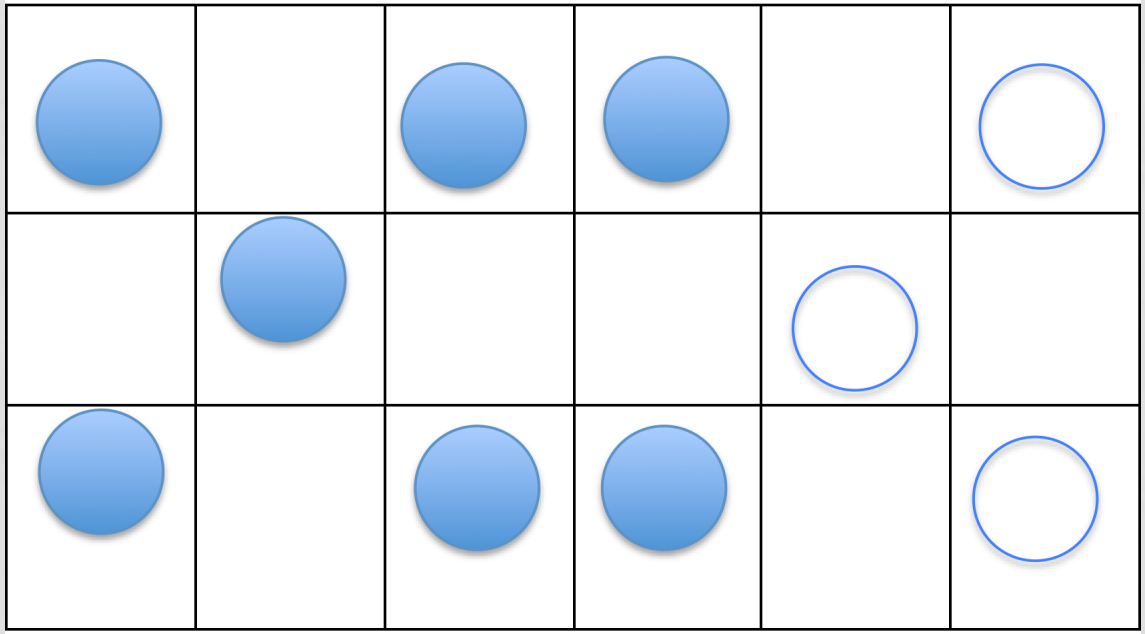
Stockholms universitet

Five rack/Ten Frame



Hungarian approach

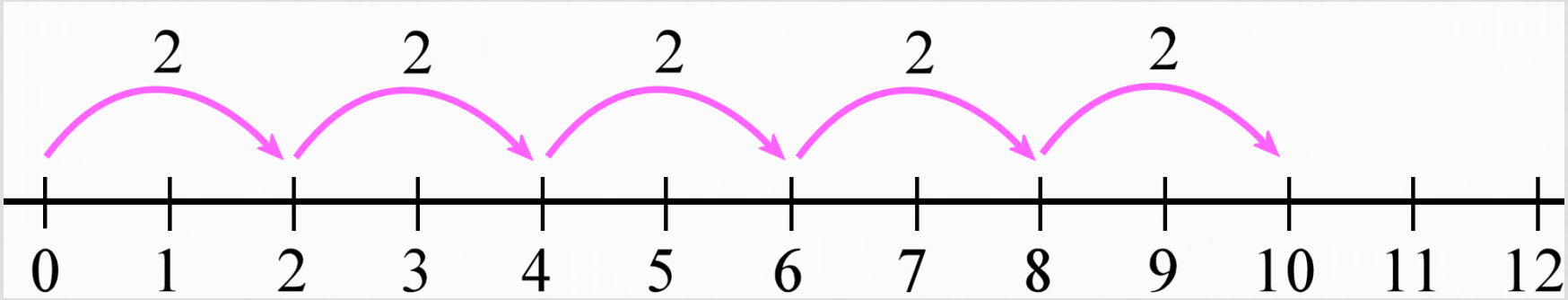


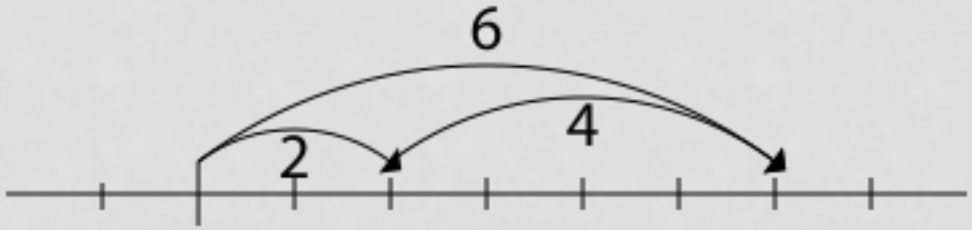


Stockholms universitet

MOVING ON FROM COUNTING…

- Counting should be short term check on subitising/ conceptual subitising moving on to skip counting and groups e.g. number line:



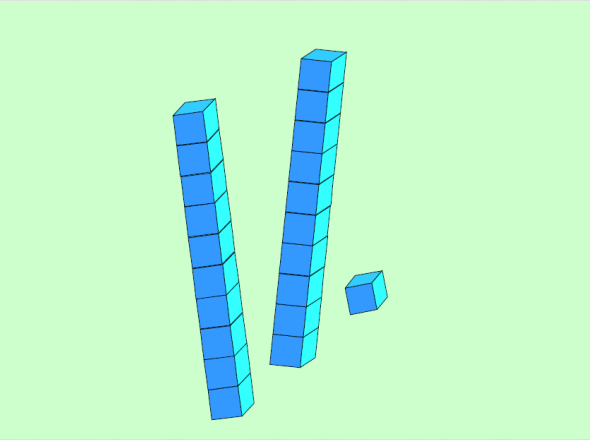


Stockholms universitet

SUPPORT INDIVIDUALS WITH QUANTITIES ALONGSIDE NUMBER LINE

Cardinality

- Manipulating quantities with:
- rods of cubes
- Bundles of straws
- Base 10 rods etc.



Operations can be modelled in different ways using different resources.

- Decomposition and composition of numbers using a variety of materials provides images to visualise and develop that shift to mental calculation

Stockholms universitet

NUMICON

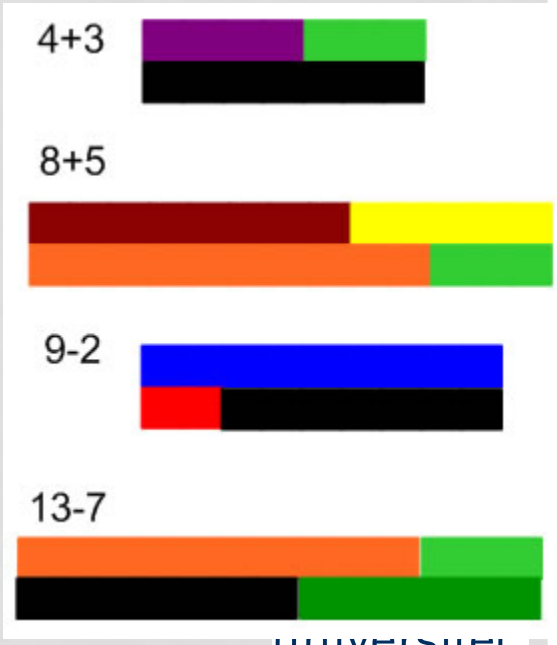
The Primary 1,2 guide to Numicon in 7 minutes
https://www.youtube.com/watch?v=EIGN3ekzpjc

Adding 2x2 digit numbers
https://www.youtube.com/watch?v=0lj3Yg1TbA8

http://www.numicon.co.nz/resources.html

Stockholms universitet

CUISENAIRE RODS

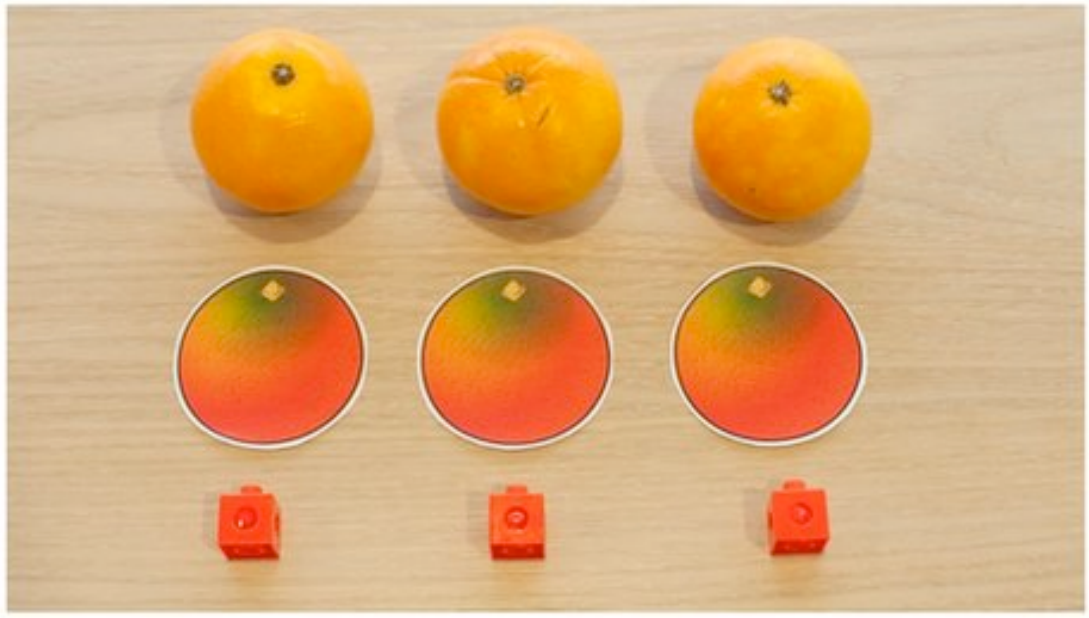


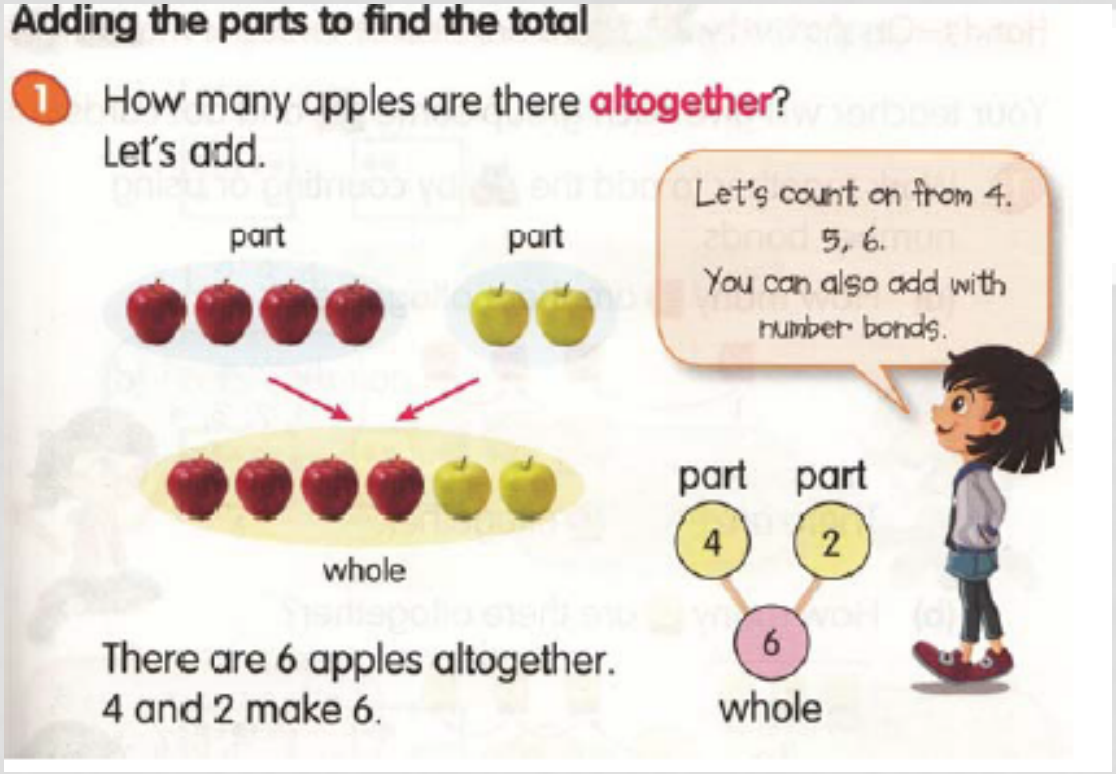
http://nrich.maths.org/public/leg.php?code=-297&cl=2&cldcmpid=4561

SINGAPORE METHODS OF TEACHING…

- The Singapore method of teaching early mathematics is to encourage and teach explicit representations. Such as the picture shows.

They work on counting and number bonds for over half a year before expecting children to engage in addition subtraction.





- Level 2

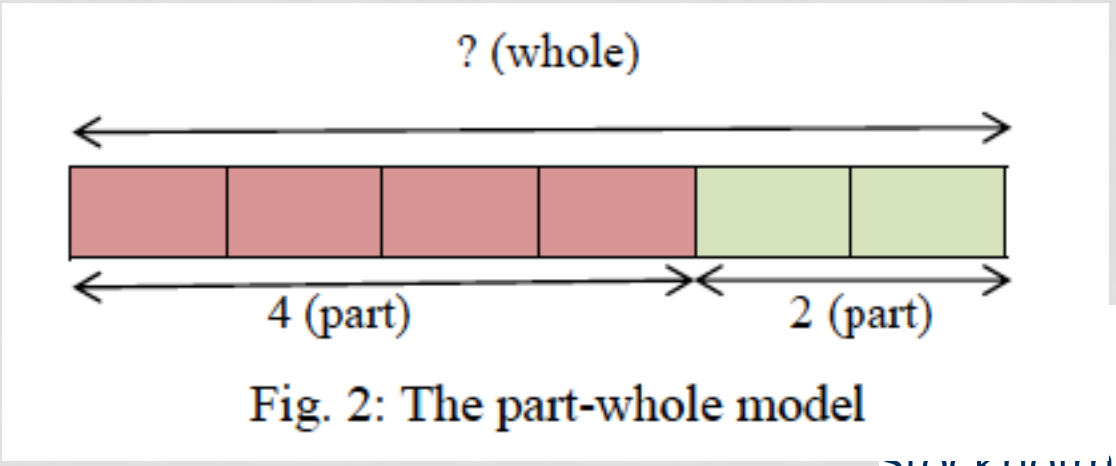


Fig. 2: The part-whole model

LEVEL 2 CONT.

- Comparison

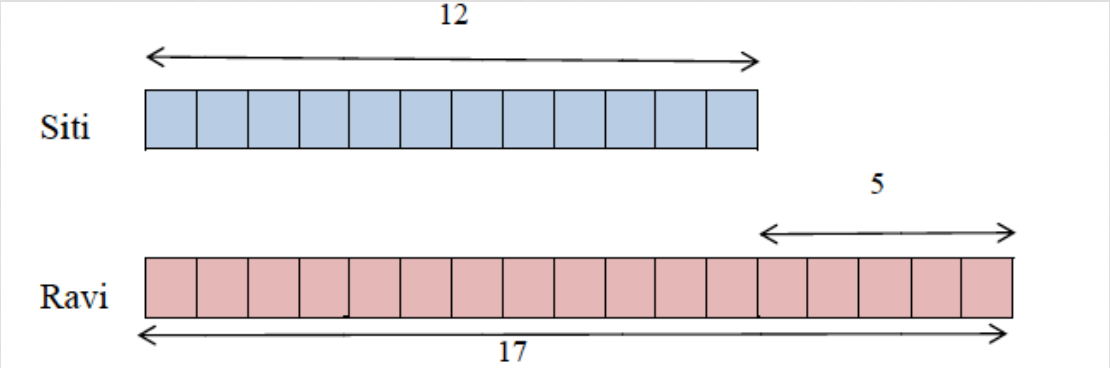
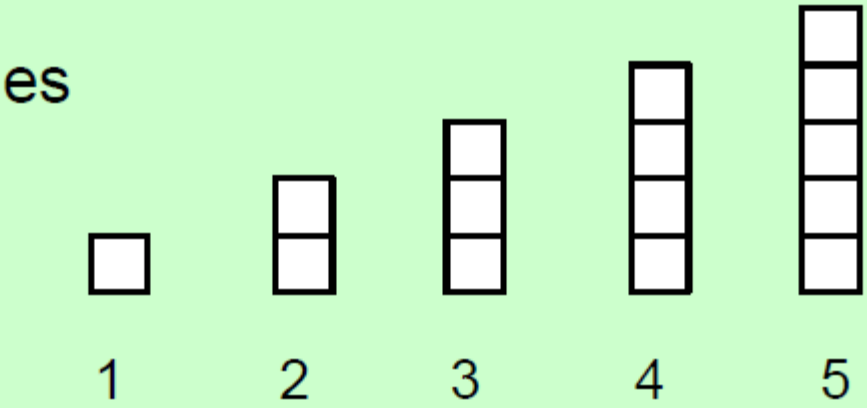


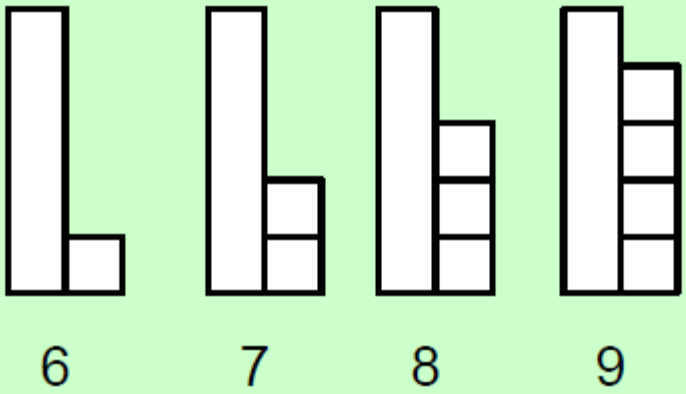
Fig. 4: The comparison model

- 'bar method'

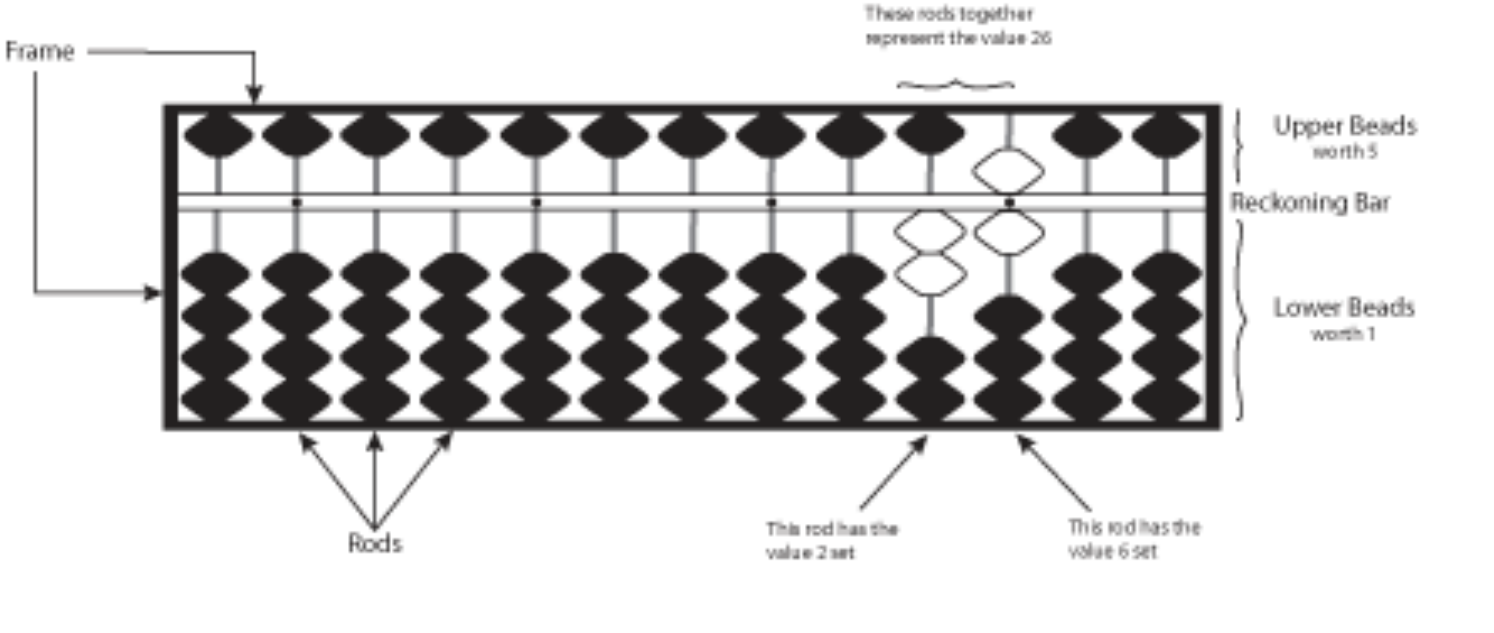
Stockholms universitet

Japanese
number tiles





SOROBAN



Stockholms universitet

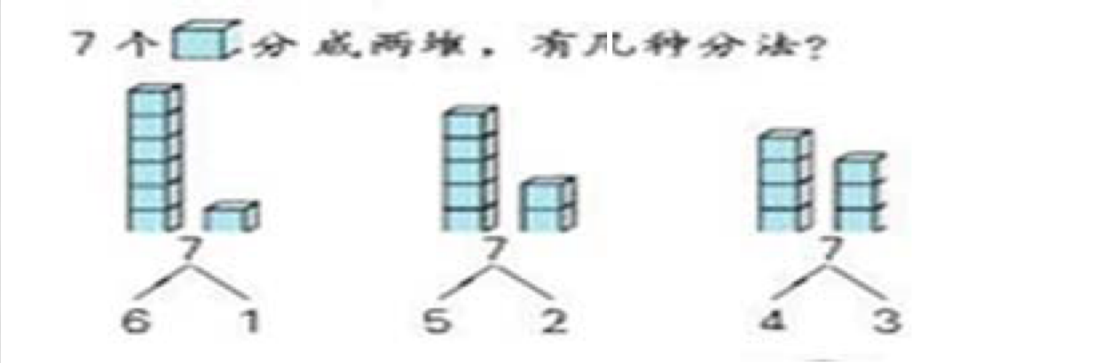
SOROBAN & SUANPAN

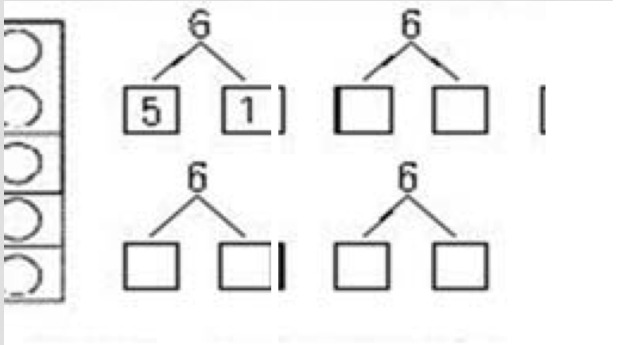
- Japanese abacus - Soroban
- http://www.ictgames.com/soroban/
- Chinese abacus - Suanpan
- http://www.alcula.com/suanpan.php
- **Emphasis on:- PLACE VALUE**

Stockholms universitet

COUNTING…

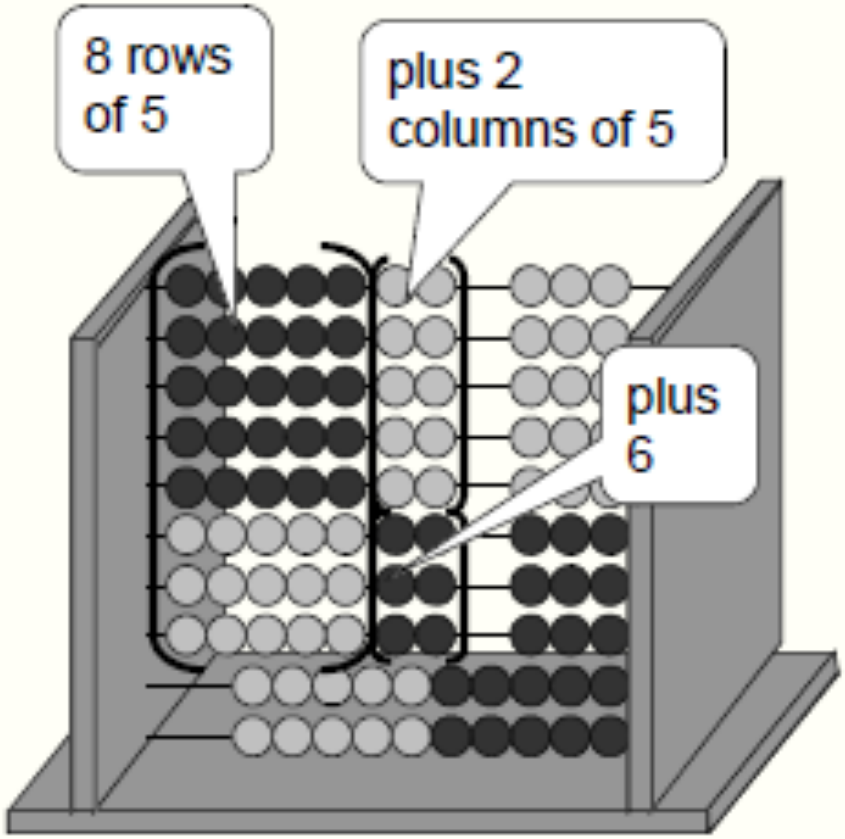
- Of course counting is a first step to calculation…





Stockholms universitet

Multiplying on the Slavonic Abacus



Eight times seven is
eight rows of five,
plus two columns of five,
plus six,
which is ten fives, plus six,
which is fifty six

https://www.youtube.com/watch?v=Wm5GsErcOUk

EINSTEIN

- ‘Everything should be made as simple as possible, but not simpler.’

Stockholms universitet

MAKING MATHEMATICS ACCESSIBLE

- Number lines (and its relation to measurement)
- 100 squares
- Bead strings
- Arrays (and dotty patterns)
- The slavonic abacus
- Soroban or Suanpan
- Cuisenaire Rods

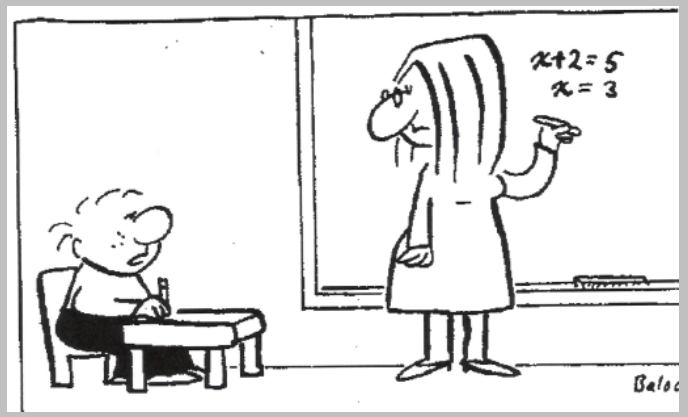
Stockholms universitet

Visual representation
Using key resources
Being comfortable, safe and secure
Sharing knowledge (peers and adults)
Individually, pairs, small groups, whole class – mix
Continuity (teachers and TAs)
Good role models
Structured environment
Behavioural strategies
Learning which relates to real life
Different teaching styles
Practical, hands-on experiences
Learning through play
Interactive environments
Discussion
Calm environment
Games

Which of these strategies and techniques have you used?

FLEXIBILITY, CONSISTENCY AND SECURITY

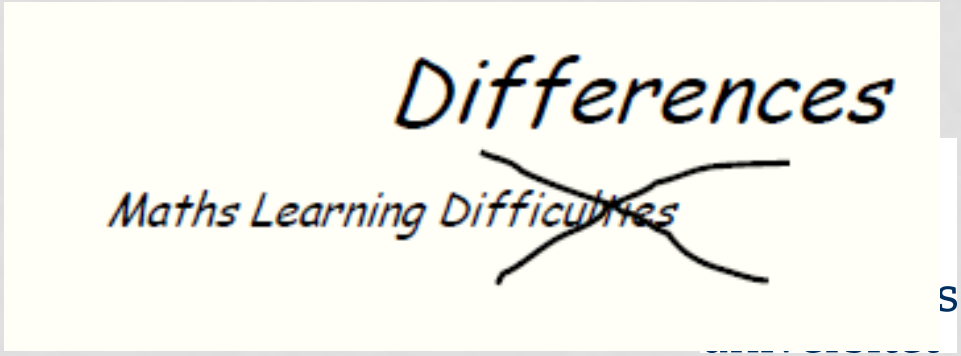
- The desire for consistency is a central motivator of our behaviour.
  - Consistency allows us a convenient, relatively effortless and efficient method for dealing with complex daily environments that make severe demands on our mental energies and capacities.'
- 'Influence (Caldini 2007)



"Just a darn minute! Yesterday you said that x equals two!"

IN SUM…

- Some children cannot express what they mean.
- As educators we must look to providing and developing their use of different representations to support children's intellectual and emotional development.
- We must provide appropriate environments for these developments to happen.



FURTHER READING

- Instrumental and relational understanding in mathematics Richard Skemp www.skemp.org.uk

The Slavonic abacus: Tandi Clausen-May

http://www.atm.org.uk/free-resources/slavonicabacus.html

Ann Dowker (2009)

http://www.atm.org.uk/free-resources/slavonicabacus.html

CONCLUSION…WHAT IS TEACHING?

- Philosophy of Education (infl. cultural)
- Learning Theory (infl. personal/institutional)
- As a teacher YOU handle…
  - Academic Development
  - Social Development
  - Emotional Development of the students in your class

- **What you do really matters!**

Stockholms universitet